Life of Fred®
Metamathematics

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Polka Dot Publishing

Contents

Chapter One
It All Begins with a Bug

Fred had his teaching schedule posted on his door.

—room 314—

8–9 Arithmetic
9–10 Beginning Algebra
10–11 Advanced Algebra
11–noon Geometry
noon–1 Trigonometry
1–2 Calculus
2–3 Statistics
3–3:05 Break
3:05–4 Linear Algebra
4–5 Seminar in Biology, Economics, Physics, Set Theory, Topology, and Metamathematics.

Wait a minute! I, your reader, have a question. I've seen this teaching schedule in a half dozen different Life of Fred books and have always wondered about that 3–3:05 Break***. That six-year-old sure seems to have a lot of stamina.***

Over the years readers have emailed me with that question. After seven hours of teaching (8 a.m. – 3 p.m.), many of us would use that five-minute break as a bathroom break. Not Fred.* Since he eats and drinks virtually nothing, his trips to the bathroom are rare.

Instead, he dashed to the Math Building, up to the third floor, past the seven vending machines (three on the left and four on the right), into his office (room 314), and sat at his desk. This gave him four minutes and 40 seconds to just sit and think.

He looked down. On his desktop was a strip of artist tape that Kingie had accidentally left there. It was divided into little squares and stretched out in both directions off the ends of the desk.

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* This is not a sentence fragment. It is called an **elliptical construction**. Words have been left out that the reader can easily insert. The full sentence might be, "It was not the case with Fred."

(Almost everyone knows who Kingie is, but if this is your first Life of Fred book, you might not know that Kingie is Fred's doll who was given to him when Fred was about four days old. Kingie is an internationally known oil painter.)

With Fred's little pointy eyes he noticed a bug on one of the squares.

How cute! Fred thought. *I wonder if it can do any tricks.*

He watched. The bug moved back and forth on the tape. Sometimes it would change the square from white to black. And sometimes it would take a black square and change it back to white.

After several moves it stopped.

Here are the facts:

① The squares can be either white or black.

② What the bug does depends on just two things: what color square he is on and what mood he is in.

③ The bug's mood can be either curious, glad, afraid, friendly, itchy. . . . (a finite list)

④ The bug has no memory. Depending on the square he is on and the mood he is in, he does three things: ❶ He colors the square he is on either white or black, ❷ He moves one square to the left or to the right, and ❸ He chooses a mood—curious, glad, afraid, friendly, itchy. . . .

For example, if the square he is on is black and his mood was curious, he might decide to color the square black, move one square to the left, and change his mood to glad.

(Black, Curious) → (Black, Left, Glad)

If he were on a black square and glad, he might decide to color the square black, move one square to the left, and change his mood to curious.

(Black, Glad) → (Black, Left, Curious)

Let's say that the bug always starts in a curious mood.

Fred thought. Bug brains! With no memory and operating under a fixed set of rules, what's the big deal?

Fred wrote out one possible bug brain if the bug has two moods:

(white, curious) → (color it black, move right, change mood to glad)
(black, curious) → (color it black, move left, change mood to glad)
(white, glad) → (black, left, curious)
(black, glad) *leave this blank*

Fred had to leave one of the instructions blank or the bug would never stop.

Your Turn to Play #1 Take out a piece of paper and follow the steps that this bug will make until it stops. What will the tape look like after those six steps?*

The one thing that Fred had control over was the content of the bug's brain.

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* Doing these *Your Turn to Play* challenges is important. You will learn a lot more than if you just passively read the story. Students will often remember the seconds when they spoke up in class rather than the hours they spent listening to lectures.

In the early Life of Fred books, I put the answers to the *Your Turn to Play* questions on the next page. In more advanced ones I put the answers in the back of the book, hoping to encourage readers to work on the problems instead of just turning to the answers.

I faced two alternatives: either supply the answers, which would encourage passivity, or withhold the answers, which would be mean.

Or I could include some of the answers, which would be an uncomfortable compromise.

In this book—are you ready?—I will include all of the answers to the *Your Turn to Play* questions, but I will bury them rather than make them hyper-convenient. Several pages from now I will write, "#1 After six steps the tape will look like . . .".

So do the work and keep it on each of the *Your Turn to Play* problems.

When Fred wrote one possible configuration of a bug with two moods:

(white, curious) → (color it black, move right, change mood to glad)
(black, curious) → (color it black, move left, change mood to glad)
(white, glad) → (black, left, curious)
(black, glad) *leave this blank*

he could have made it into a chart:

	Curious	Glad
White	color it Black, move Right, become Glad	color it Black move Left become Curious
Black	color it Black, move Left, become Glad	leave blank so the bug will halt

or more simply:

	Curious	Glad
White	B, R, G	B, L, C
Black	B, L, G	leave blank

or: (W, C) → (B, R, G)
(B, C) → (B, L, G)
(W, G) → (B, L, C)
(B, G) left blank

You now have four different ways to describe a bug's brain. Any of them are fine. Take your choice.

Some notes:

♪#1: For a bug with two moods, Fred had many choices:

(W, C) → (?, ?, ?)
(B, C) → (?, ?, ?)
(W, G) → (?, ?, ?)
(B, G) left blank

He had nine spots to fill in.

If he had written
(W, C) → left blank
(B, C) → (?, ?, ?)
(W, G) → (?, ?, ?)
(B, G) → (?, ?, ?)

the bug would never move at all.

Your Turn to Play #2 If Fred had written (W, C) → (W, R, C) what would the journey of the bug look like?

♪#2: Fred's (W, C) → (B, R, G)
(B, C) → (B, L, G)
(W, G) → (B, L, C)
(B, G) left blank was special. It was the longest running program for a two-mood bug.* The bug would take six steps before it halted. No two-mood bug can ever go seven steps and halt.

For programs that halt . . .

a bug with one mood can go at most 1 step
a bug with two moods can go at most 6 steps
a bug with three moods can go at most 14 steps.

To find the bug brain that will go 14 steps, all you have to do is fill in . . .

	Curious	Glad	Afraid
White	?, ?, ?	?, ?, ?	?, ?, ?
Black	?, ?, ?	?, ?, ?	?, ?, ?

where one of the ?, ?, ? is blank. My university teacher gave this as a homework assignment. I spent all night trying to get that bug to hop 14 times. This was insanely difficult. Even to write a computer program to try out various options was beyond me. I certainly would not make this a *Your Turn to Play* problem!

♪#3: Finding the chart to produce the longest running bug for different number of moods is called the **busy beaver function**.
1 mood → one step
2 moods → 6 steps
3 moods → 14 steps

* Not counting the programs that run forever.

4 moods → 107 steps
5 moods → No one on planet earth (as of this writing) knows! Heiner Marxen and Jürgen Buntrock found a chart back in 1989 that made the bug move 47,176,870 steps before stopping. So we know that for five moods, there are at least 47,176,870 steps.

This bug deserves some respect.

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